General Instructions to Candidates:

- There is a ‘Cool-off time’ of 15 minutes in addition to the writing time of 2 ½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the ‘cool-off time’.
- Use the ‘cool-off time’ to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

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1. (a) Give an example of a relation on a set \( A = \{1, 2, 3, 4\} \) which is reflexive and symmetric but not transitive. \( \quad \) (1)

(b) Show that \( f : [-1, 1] \rightarrow \mathbb{R} \) given by \( f(x) = \frac{x}{x+2} \) is one-one. \( \quad \) (2)

(c) Let \( * \) be a binary operation on \( Q^+ \) defined by \( a * b = \frac{ab}{6} \). Find the inverse of 9 with respect to \( * \). \( \quad \) (2)

2. (a) The principal value of \( \tan^{-1}(1) \) is .... \( \quad \) (1)

(b) Write the function \( \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) \), where \( x \neq 0 \) in the simplest form. \( \quad \) (3)

3. Consider a \( 2 \times 2 \) matrix \( A = [a_{ij}] \), where \( a_{ij} = |2i - 3j| \).

(a) Write \( A \). \( \quad \) (2)

(b) Find \( A + A' \). \( \quad \) (1)

4. (a) Find the value of \( k \) if \( f(x) = kx^2 \), \( x \leq 2 \)

\[ = 3, \quad x > 2 \]

is continuous. \( \quad \) (2)

(b) Find \( \frac{dy}{dx} \) if \( y = x \sin x, \ x > 0 \). \( \quad \) (2)

1018
(c) If \( y = \sin^{-1} x \), show that \( (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0 \). \hspace{1cm} (2)

5. Consider the matrix \( A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \).
   (a) Find \( A^2 \). \hspace{1cm} (2)
   (b) Find ‘k’ so that \( A^2 = kA - 7I \). \hspace{1cm} (1)

6. (a) The slope of the tangent to the curve \( y = x^3 - 1 \) at \( x = 2 \) is ... \hspace{1cm} (1)
   (b) Use differential to approximate \( \sqrt{36.6} \). \hspace{1cm} (2)
   (c) Find two numbers whose sum is 24 and whose product is as large as possible. \hspace{1cm} (2)

7. (a) If \( \begin{vmatrix} x & 3 \\ 5 & 2 \end{vmatrix} = 5 \), then \( x = ... \). \hspace{1cm} (1)
   (b) Prove that \( \begin{vmatrix} y + k & y & y \\ y & y + k & y \\ y & y & y + k \end{vmatrix} = k^2 (3y + k) \). \hspace{1cm} (2)
   (c) Solve the following system of linear equations, using matrix method:
   \[ 5x + 2y = 3 \]
   \[ 3x + 2y = 5 \]. \hspace{1cm} (2)

1018

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8. (a) \(\int \tan x \, dx = \ldots\) (1)

(b) Evaluate \(\int \sin^2 (2x + 5) \, dx\). (2)

(c) Evaluate \(\int \frac{x \, dx}{(x + 1)(x + 2)}\). (2)

9. Consider the points A(1, 2, 3), B(4, 0, 4) and C(−2, 4, 2).

(a) Find \(\overrightarrow{AB}\) and \(\overrightarrow{BC}\). (2)

(b) Show that the points A, B, C are collinear. (1)

10. (a) If \(f(x) = \int_0^x t \sin t \, dt\), then

\(f'(x) = \ldots\) (1)

(b) Evaluate \(\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} \, dx\). (4)

11. Consider the vectors \(\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}\)

and \(\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}\).

(a) Find \(\vec{a} \cdot \vec{b}\) and \(\vec{a} \times \vec{b}\). (3)

(b) Verify that \(|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \ |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2\). (2)
12. (a) Find the area of the region bounded by the curves \( y^2 = x \) and the lines \( x = 1, x = 4 \) and the \( x \)-axis. (2)

(b) Using integration, find the area of the triangle with vertices (1, 0), (2, 2) and (3, 1). (4)

13. Consider the points \( A(3, -4, -5) \) and \( B(2, -3, 1) \).

(a) Find the vector and Cartesian equations of the line passing through the points \( A \) and \( B \). (2)

(b) Find the point where the line crosses the \( XY \) plane. (2)

14. Consider the differential equation \( x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx \).

(a) Express it in the form \( \frac{dy}{dx} = F(x, y) \). (1)

(b) Show that it is homogeneous of degree zero. (2)

(c) Find its general solution. (3)

15. (a) Find the Cartesian equation of the plane passing through the point (1, 2, -3) and perpendicular to the vector \( 2\hat{i} - \hat{j} + 2\hat{k} \). (2)

12. (a) \( y^2 = x \) where \( x = 1, x = 4, x-\text{axis} \) are the curves forming the area bounded by the \( x \)-axis and the \( x \)-coordinates of the points intersecting the curve.

(b) (1, 0), (2, 2), (3, 1) are the points forming the triangle with vertices (1, 0), (2, 2) and (3, 1).

13. (a) \( A(3, -4, -5), B(2, -3, 1) \) are the points forming the triangle.

(a) \( A, B \) are the points forming the triangle.

(b) Show that it is homogeneous of degree zero.

(c) Find its general solution.

14. \( x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx \) is the differential equation.

(a) \( \frac{dy}{dx} = F(x, y) \) is the general solution.

(b) Show that it is homogeneous of degree zero.

(c) Find its general solution.

15. (a) \( 2\hat{i} - \hat{j} + 2\hat{k} \) are the coefficients forming the equation of the plane passing through the point (1, 2, -3).

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(b) Find the angle between the above plane and the line $\frac{x-1}{2}$
$$\frac{y-3}{3} = \frac{z}{6}.$$  \hspace{1cm} (2)

16. Consider the linear programming problem:

Maximize $Z = x + y$
Subject to $2x + y - 3 \geq 0$
$x - 2y + 1 \leq 0$
$y \leq 3$
$x \geq 0, y \geq 0$

(a) Draw its feasible region.  \hspace{1cm} (3)
(b) Find the corner points of the feasible region.  \hspace{1cm} (2)
(c) Find the corner at which $Z$ attains its maximum.  \hspace{1cm} (1)

17. (a) Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that one of them is black and other is red.  \hspace{1cm} (2)

(b) Find the probability of getting 5 exactly twice in 7 throws of a die.  \hspace{1cm} (3)

OR

6
(a) A die is tossed thrice. Find the probability of getting an odd number at least once.  

(2)

(b) Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.  

(3)